Circling the Square Quad

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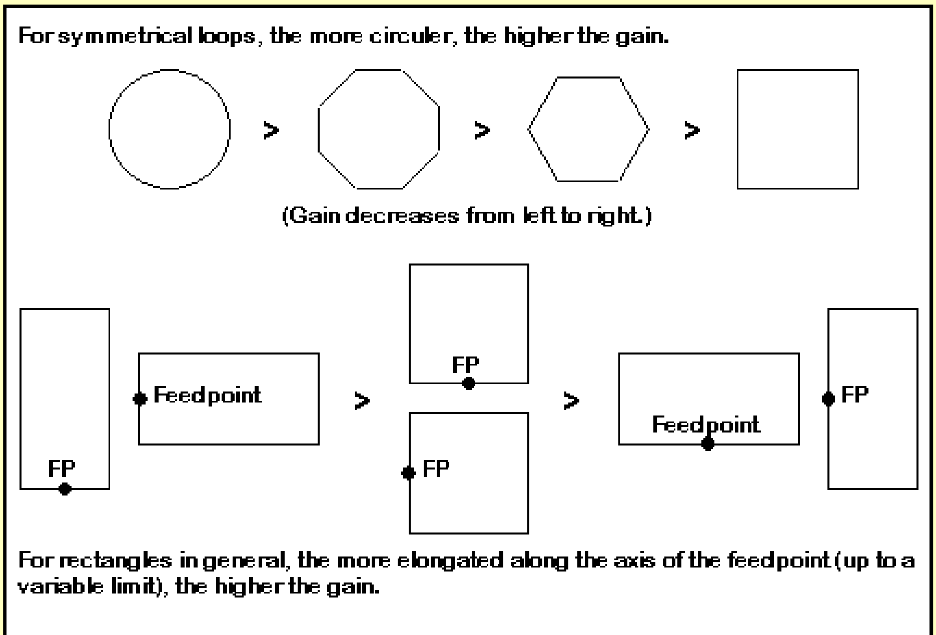
A question posed to me every now and then is whether the circumference of a circular quad loop and the circumference of a square quad loop--both with the same diameter wire and the same resonant frequency--are the same. It is an interesting question, since without any analysis, it seems to have a simple answer. However, like most questions of its type, it hides numerous complexities, some related to the motivation for the question, others related to the variables involved in developing some kind of reliable answer.

Setting the Circular Loop in Its Context

Since the most common form of quad loop used by VHF operators is square, one must wonder why the urgency to turn to a circular loop. The square loop normally uses 4 non-conductive spokes to support the corners of a wire square, where the wire may run from AWG #14 copper to about 0.25" thin-wall tubing. At 2-meters (146 MHz, as a target frequency), this diameter range runs from 0.0031 wavelength to 0.00079 wavelength, a 4:1 ratio between the largest and smallest diameters in the set. Smaller wires present the fewest construction complications, although they show the smallest operating bandwidth for a given quad design.

Circular loops, on the other hand, challenge the builder. There are some commercial 2-meter quads using flat strap loops in order to obtain a round shape with only 1 mounting point to a boom. However, we shall have to confine ourselves in these notes to round wires, which do not enter a perfectly round shape--or sustain it--with great ease. Hence, we once more must inquire into the motivation for building a quad array with circular loops.

Among novice antenna builders, there lurks a danger: learning something as a one-line generalization. The applicable general statement most often quoted to me is that a circular loop has the highest gain possible, higher than a square loop. Hence, it makes sense to strive for a circular-loop quad rather than settling for a square-loop quad. There are only 2 difficulties opened by this generalization.



Circularity as a Variable in the Loop-Gain Picture Fig. 1

First, as portrayed in **Fig. 1**, the generalization is incomplete in its context. As shown in the top portion of the figure, it is more proper to say that among symmetrical loops, the circular shape has the highest gain of all of the possible polygon substitutes. The greater the departure from circularity, the lower the gain, if we maintain a regular polygon shape.

The lower half of the figure expands the context. It shows the symmetrical or regular polygon in the center--with a feedpoint shown--between two alternative configurations, one with higher gain and one with lower gain. As shown in numerous sources, as we stretch the axis between the feedpoint and its opposite side--maintaining resonance throughout--the rectangle increases in gain as those sides approach a separation of 1/2 wavelength. There are limits to this process. One is physical: we cannot make a rectangle that is 1/2 wavelength long and still have a resonant rectangle. A second reason emerges from the properties of the necessary wire implementation of the rectangle. Wire losses will set a point, one that varies with wire diameter and composition to the shape of the rectangle, that yields the highest gain. A final consideration is the feedpoint impedance that decreases as we stretch the rectangle to increase gain. At some point, the resonant impedance becomes too low for effective use.

At the other end of the figure are rectangles fed along their longer sides. These rectangles show lower gain and higher feedpoint impedances than symmetrical polygons. The stretched rectangles with feedpoints on the short side, especially vertically polarized versions, are widely used in the lower HF region, to obtain gain over simple vertical antennas in a bi-directional pattern. They require construction techniques that fall within the abilities of many amateur antenna builders and so have developed a niche for themselves.

The stretched rectangle offers a considerable gain advantage over a square loop, about 1 dB for versions presenting a feedpoint impedance close to 50 Ohms. (A resonant square loop has a feedpoint impedance in the 120-130-Ohm range, depending upon wire size.) What we are missing in the case of comparing a circular loop to a square loop is a quantification of the amount of gain advantage that it has over a square. As well, we have no translation of the gain of a single circular loop over a square relative to the gain advantage of a circular-loop quad beam over its counterpart square-loop version.

As well, we have no easy answer to the question of whether a circular loop having the same circumference as a counterpart square loop will also have the same resonant frequency. If not, we shall need an adjustment factor. This factor will normally be relative to the loop circumference of a square, since most available quad designs in the literature use square loops.

I have in the past suggested an empirical method of determining the adjustment factor. At 2-meters (146 MHz will be our design frequency throughout this exercise), build a single resonant square quad loop. Then, using the same wire, build a circular loop, adjusting its circumference for resonance at the same frequency as the square. The ratio of the two circumferences will be the adjustment factor for all of the elements in the array being rounded--if there are no other design variables to consider.

A modeling approach to the question is necessarily limited, since NEC and MININEC models must use straight-wires to form a polygon that approaches circularity. However, even within this limitation, modeling offers some advantages over the empirical approach, since we may use as our subject antennas arrays of some complexity. Remember that part of our question involves--beyond a simple adjustment factor for loop circumference--an inquiry into whether there may be additional design considerations in the conversion. In other words, will a circular-loop beam have the same operating characteristics as its square counterpart across a given operating passband?

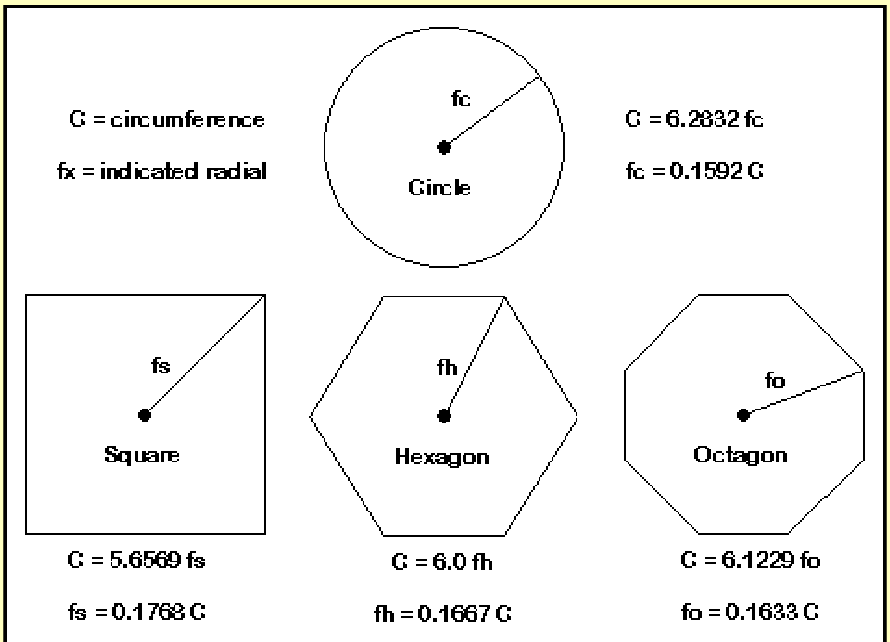
Arbitrarily--although based on some experience in designing quad beams--I selected a 3-element quad beam optimized for maximum gain at the center of the 2-meter band as a worthy test case. The use of a high gain design means that the gain, front-to-back, and impedance curves will be sharp enough to observe any shifts between square and near-circular versions. As well, a 3-element beam would provide an indication of the gain advantage of going circular, relative to the square, and do so in a context that would preclude a common error. If we developed a gain advantage for a single loop over a square, then some novice antenna buffs would simply add that gain for each new loop appended to the array. Unfortunately, antennas tend not to work that way. A geometry-based gain increase occurs once, and that advantage tends to be the total that we may gain for a parasitic array of any complexity.

Now for a limitation: we shall not be able to model a circle. However, we may model both a hexagon and an octagon. In this way, we may approach the situation with a circle, and the trends developed by modeling both a hexagon and an octagon as the loops for a 3-element quad beam may be of some assistance in extrapolating values for a circle.

Some Basic Geometry of Loops

On the road from a square to a circle, we shall initially wish to develop quad beams having the same loop circumferences for all versions. Then, by adjusting the loop sizes, we may determine the required hexagon and octagon loop circumferences needed. We may next calculate an adjustment factor, and possibly even extrapolate to a reasonably reliable adjustment factor for circular loops.

The first step in the process is to understand something about the geometries of our subject loop shapes. **Fig. 2** summarizes the results of a little exercise in trigonometry.



How Well Polygons Approximate a Circle Fig. 2

We know that the circumference of a circle is 2 times PI times the radius of the circle, and that the inverse lets us compute the radius from the circumference. The figure gives us the final numbers to 4 decimal places.

We may use simple trig relationships to find the relationship between the circumference of almost any polygon and a focal line, if we first determine what we shall call the focal line. The simplest line runs from the polygon center to a corner--the fx lines for the 3 shapes shown. We shall use the same type of focal line in each case for consistency.

For the case of the square, the line designated fs is 1/2 the square root of 2 (that is, 0.5 SQR(2)) times the length of a side, which is 1/4 of the total circumference. That means the circumference is the given value in the sketch relative to a focal line, fs, of any length.

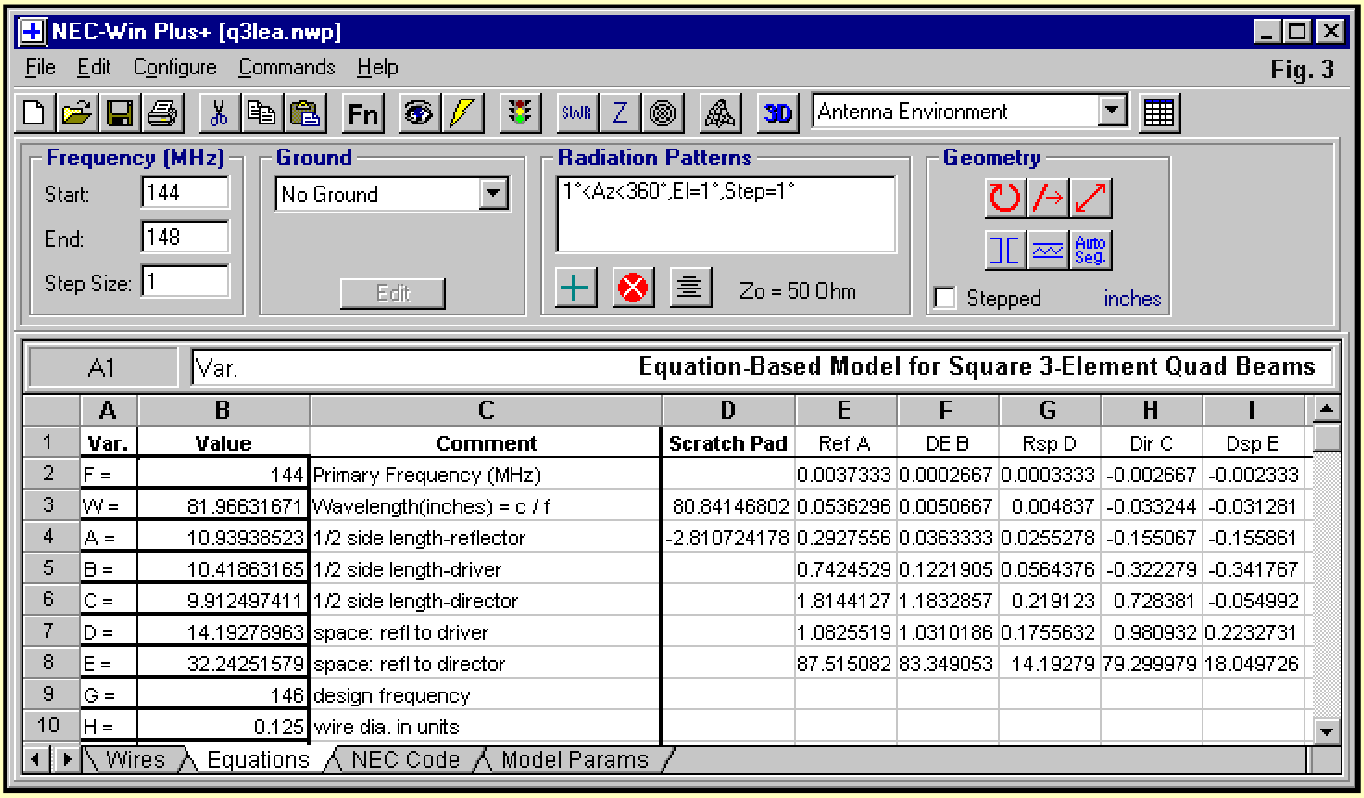
The hexagon is the easy case, since we can build one by using a collection of equilateral triangles. Hence, the focal line, fh, is the same length as any one side. The circumference is 6 times the length of fh.

The octagon seems more complex simply because we are not used to working with 22.5-degree angles. However, 1/2 the length of any side is the sine of 22.5 degrees times the length of the focal line, fo. Once we have the length of a half-side, we multiply by 16 to obtain the circumference.

Notice that the relationship between the circumference and focal line of the octagon is rapidly approaching the relationship between the circumference and radius of a circle. For many purposes, modelers often use octagons as adequate approximations of a circle. However, we shall not stop with the octagon. We shall use the trends developed by going through the hexagon to see if we cannot approximate what a circular loop should use as its circumference. The result will not be precise, because our steps from the square to the circle--in terms of the ratio of C to fx--are not regular. As well, our modeling will itself be subject to some limitations of precision. Nevertheless, we may be able to produce a reasonable value for the backyard builder, along with some other considerations for him to evaluate.

A Matter of Process

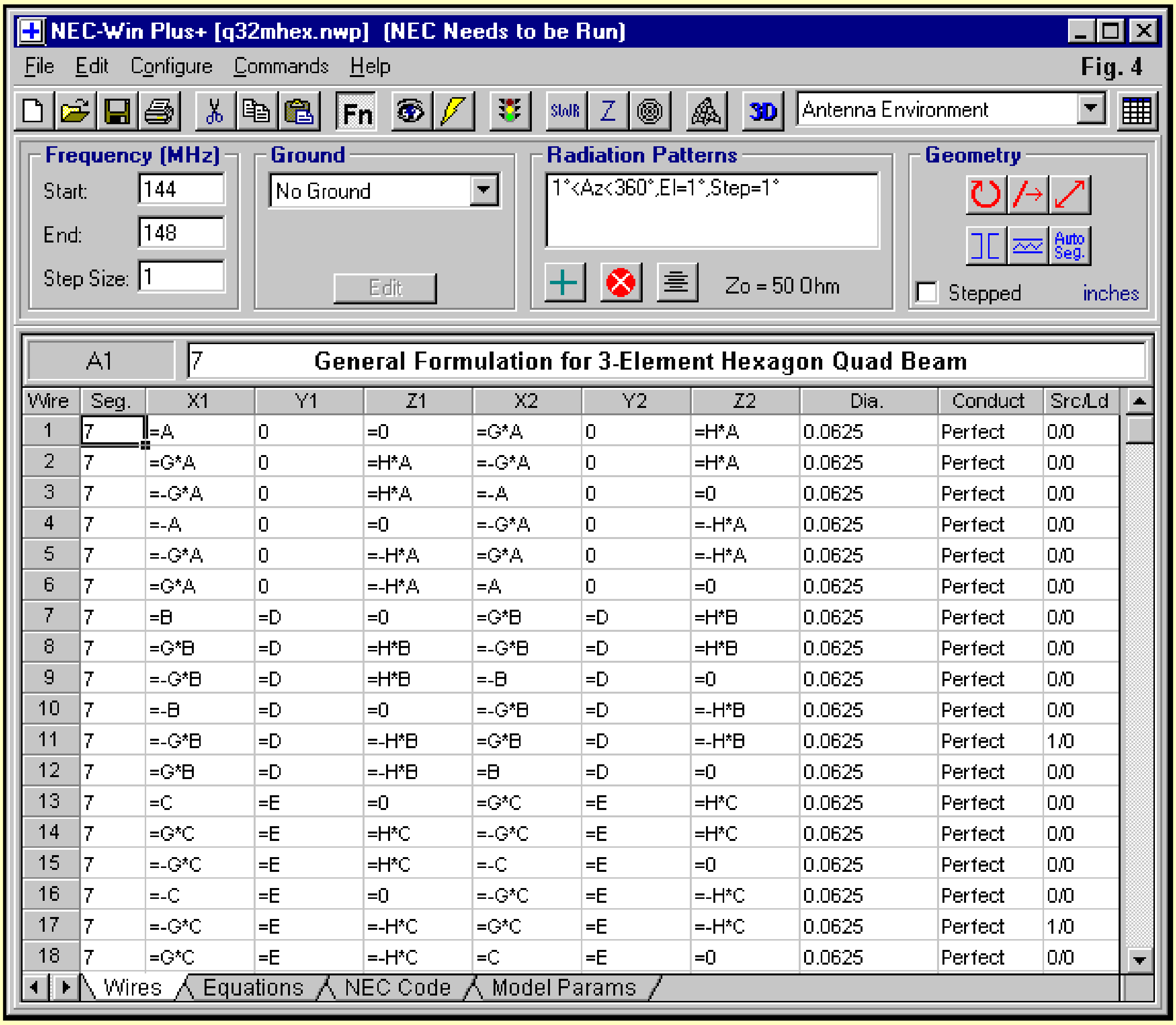
To start on our process, we need a source of square quad designs. I used the NEC-Win Plus equation-based model for a 3-element "high gain" quad (in contrast with a "wide-band" design using the same number of elements). The model is available from the Nittany Scientific web site (http://www.nittany-scientific.com), and a simple calculating program in Windows form can be downloaded from the web site under "New Quad Studies" (../quad/quadlist.html).



	A	B	C	D	E	F	G	H	I
1	Var.	Value	Comment	Scratch Pad	Ref A	DE B	Rsp D	Dir C	Dsp E
2	F =	144	Primary Frequency (MHz)		0.0037333	0.0002667	0.0003333	-0.002667	-0.002333
3	W =	81.96631671	Wavelength(inches) = c / f	80.84146802	0.0536296	0.0050667	0.004837	-0.033244	-0.031281
4	A =	10.93938523	1/2 side length-reflector	-2.810724178	0.2927556	0.0363333	0.0255278	-0.155067	-0.155861
5	B =	10.41863165	1/2 side length-driver		0.7424529	0.1221905	0.0564376	-0.322279	-0.341767
6	C =	9.912497411	1/2 side length-director		1.8144127	1.1832857	0.219123	0.728381	-0.054992
7	D =	14.19278963	space: refl to driver		1.0825519	1.0310186	0.1755632	0.980932	0.2232731
8	E =	32.24251579	space: refl to director		87.515082	83.349053	14.19279	79.299979	18.049726
9	G =	146	design frequency						
10	H =	0.125	wire dia. in units						

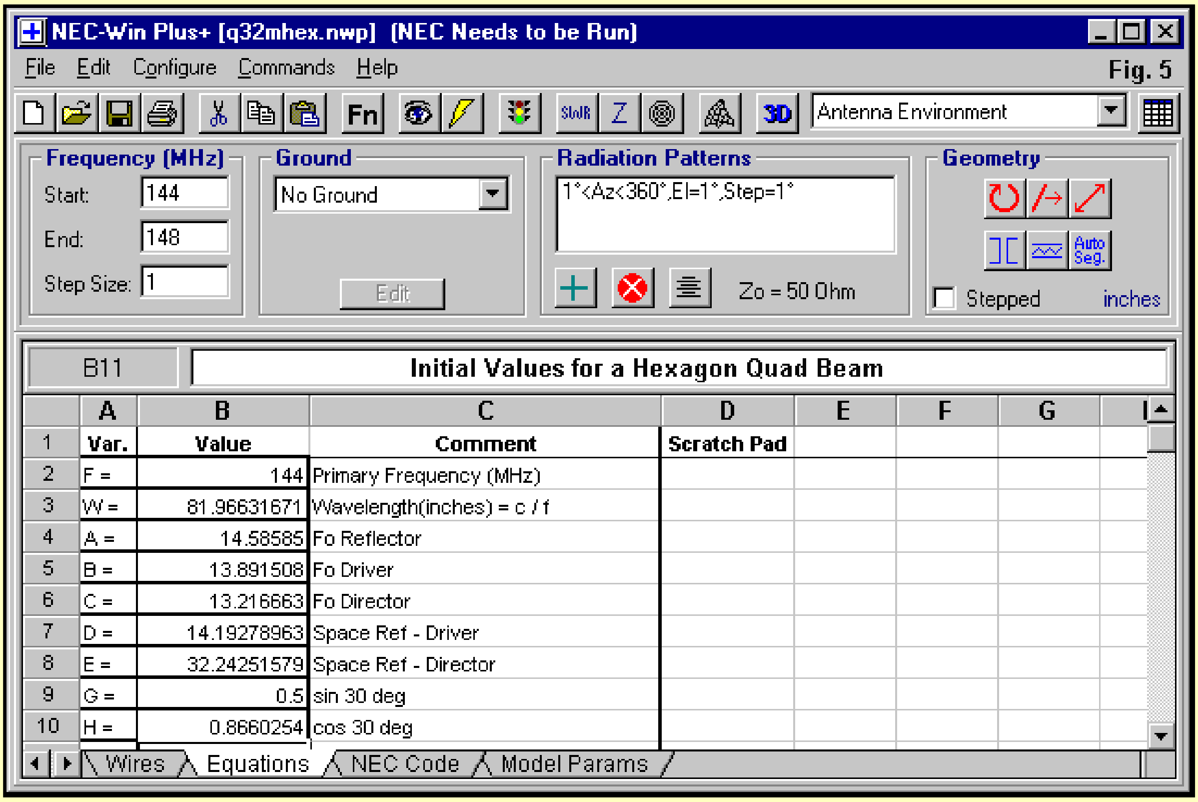
Fig. 3 shows the equations screen for the model for a square 146-MHz quad using 0.125" diameter elements. This model requires only the revision of the variables for the wire diameter in the units of measure selected for the model and the design frequency (variables G and H). The remaining variables are calculated to yield half-side lengths and reflector to driver or director spacing for use in the model itself.

By multiplying the side half-lengths times 8, we obtain the circumference of each element. We may then use the relationships in **Fig. 2** to obtain the focal line values for either hexagon or octagon versions of the model having loops with the same initial circumference. However, each type of model requires its own set-up.



Wire	Seg.	X1	Y1	Z1	X2	Y2	Z2	Dia.	Conduct	Src/Ld
1	7	=A	0	=0	=G*A	0	=H*A	0.0625	Perfect	0/0
2	7	=G*A	0	=H*A	=-G*A	0	=H*A	0.0625	Perfect	0/0
3	7	=-G*A	0	=H*A	=-A	0	=0	0.0625	Perfect	0/0
4	7	=-A	0	=0	=-G*A	0	=-H*A	0.0625	Perfect	0/0
5	7	=-G*A	0	=-H*A	=G*A	0	=-H*A	0.0625	Perfect	0/0
6	7	=G*A	0	=-H*A	=A	0	=0	0.0625	Perfect	0/0
7	7	=B	=D	=0	=G*B	=D	=H*B	0.0625	Perfect	0/0
8	7	=G*B	=D	=H*B	=-G*B	=D	=H*B	0.0625	Perfect	0/0
9	7	=-G*B	=D	=H*B	=-B	=D	=0	0.0625	Perfect	0/0
10	7	=-B	=D	=0	=-G*B	=D	=-H*B	0.0625	Perfect	0/0
11	7	=-G*B	=D	=-H*B	=G*B	=D	=-H*B	0.0625	Perfect	1/0
12	7	=G*B	=D	=-H*B	=B	=D	=0	0.0625	Perfect	0/0
13	7	=C	=E	=0	=G*C	=E	=H*C	0.0625	Perfect	0/0
14	7	=G*C	=E	=H*C	=-G*C	=E	=H*C	0.0625	Perfect	0/0
15	7	=-G*C	=E	=H*C	=-C	=E	=0	0.0625	Perfect	0/0
16	7	=-C	=E	=0	=-G*C	=E	=-H*C	0.0625	Perfect	0/0
17	7	=-G*C	=E	=-H*C	=G*C	=E	=-H*C	0.0625	Perfect	1/0
18	7	=G*C	=E	=-H*C	=C	=E	=0	0.0625	Perfect	0/0

Fig. 4 shows the basic set-up for a hexagon model. Each loop has two end point at Z=0 and X = fh. The upper and lower points on the perimeter of the hexagon require an X-value that is the sine of 30 degrees and a Z-value that is the cosine of 30 degrees, with +/- signs as appropriate to the position of a point relative to the central or boom line (Y-axis). **Fig. 5** shows the variables on the equations page. One might more fully automate the process, but this method was simple enough for our test cases.



	A	B	C	D	E	F	G
1	Var.	Value	Comment	Scratch Pad			
2	F =	144	Primary Frequency (MHz)				
3	W =	81.96631671	Wavelength(inches) = c / f				
4	A =	14.58585	Fo Reflector				
5	B =	13.891508	Fo Driver				
6	C =	13.216663	Fo Director				
7	D =	14.19278963	Space Ref - Driver				
8	E =	32.24251579	Space Ref - Director				
9	G =	0.5	sin 30 deg				
10	H =	0.8660254	cos 30 deg				

The hexagon array requires 18 wires for a 3-element quad, compared to only 12 for the square-loop version. Each side of the square quad used 11 segments, and each loop has 44 segments. A comparable number of segments per side for the hex is 7, resulting in 42 segments per loop. The use of an odd number of segments on each side ensures that the feedpoint segment is exactly centered on one side wire of the driver loop. The octagon loops use 5 segments per wire, for a total of 40 per loop. The total number of wires in the octagon model grows to 24.

Wires

Wire Edit Other

Fig. 6

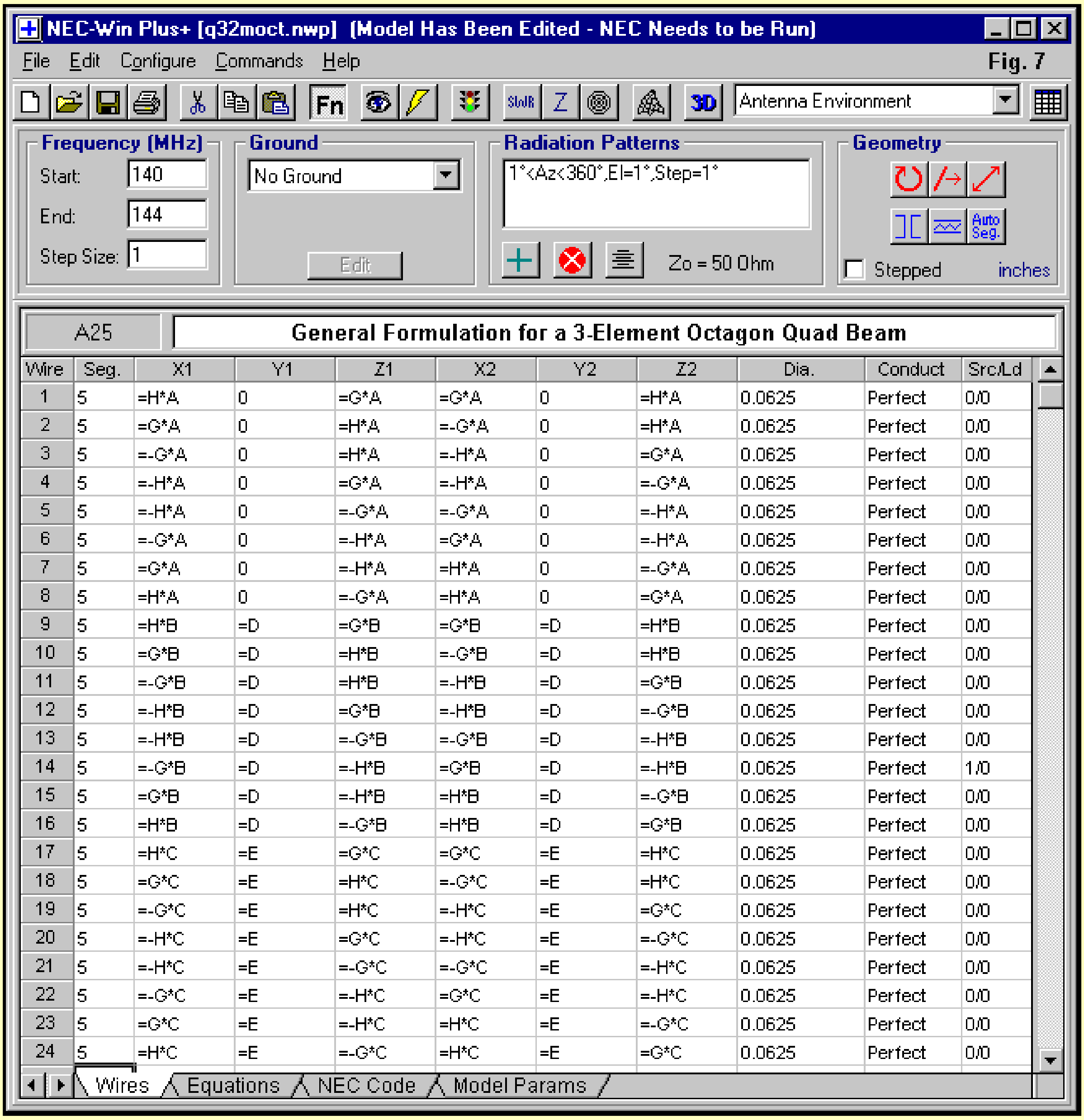
☐ Coord Entry Mode ☐ Preserve Connections

Final Values for a Hexagon Quad Beam

No.	End 1 X (in)	End 1 Y (in)	End 1 Z (in)	End 1 Conn	End 2 X (in)	End 2 Y (in)	End 2 Z (in)	End 2 Conn	Diameter (in)	Segs
1	14.2336	0	0	W6E2	7.11682	0	12.3267	W2E1	0.0625	7
2	7.11682	0	12.3267	W1E2	-7.11682	0	12.3267	W3E1	0.0625	7
3	-7.11682	0	12.3267	W2E2	-14.2336	0	0	W4E1	0.0625	7
4	-14.2336	0	0	W3E2	-7.11682	0	-12.3267	W5E1	0.0625	7
5	-7.11682	0	-12.3267	W4E2	7.11682	0	-12.3267	W6E1	0.0625	7
6	7.11682	0	-12.3267	W5E2	14.2336	0	0	W1E1	0.0625	7
7	13.6268	14.244	0	W12E2	6.81342	14.244	11.8012	W8E1	0.0625	7
8	6.81342	14.244	11.8012	W7E2	-6.81342	14.244	11.8012	W9E1	0.0625	7
9	-6.81342	14.244	11.8012	W8E2	-13.6268	14.244	0	W10E1	0.0625	7
10	-13.6268	14.244	0	W9E2	-6.81342	14.244	-11.8012	W11E1	0.0625	7
11	-6.81342	14.244	-11.8012	W10E2	6.81342	14.244	-11.8012	W12E1	0.0625	7
12	6.81342	14.244	-11.8012	W11E2	13.6268	14.244	0	W7E1	0.0625	7
13	13.019	32.2773	0	W18E2	6.50948	32.2773	11.2747	W14E1	0.0625	7
14	6.50948	32.2773	11.2747	W13E2	-6.50948	32.2773	11.2747	W15E1	0.0625	7
15	-6.50948	32.2773	11.2747	W14E2	-13.019	32.2773	0	W16E1	0.0625	7
16	-13.019	32.2773	0	W15E2	-6.50948	32.2773	-11.2747	W17E1	0.0625	7
17	-6.50948	32.2773	-11.2747	W16E2	6.50948	32.2773	-11.2747	W18E1	0.0625	7
18	6.50948	32.2773	-11.2747	W17E2	13.019	32.2773	0	W13E1	0.0625	7
*										

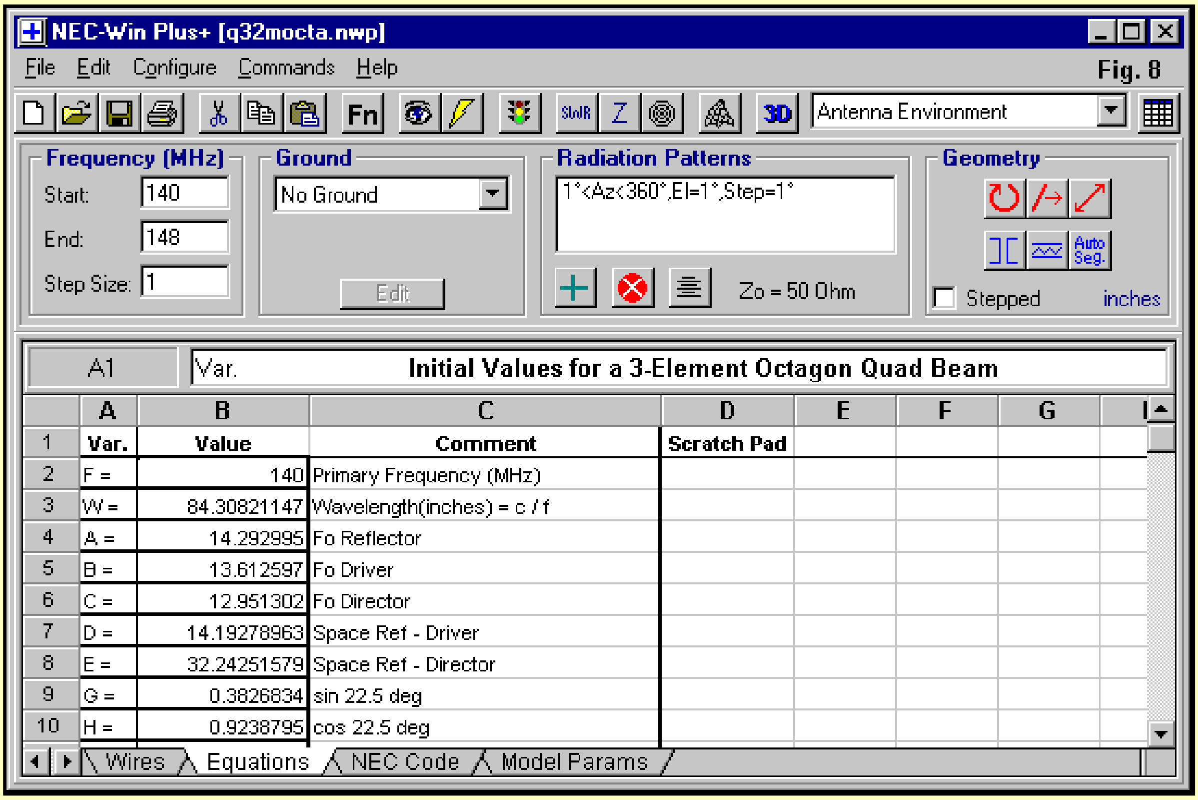
However, before moving to the octagon model, let's note in advance that the converted models do not resonate at 146 MHz, despite using loops with the same wire diameter, circumference length, and element spacing as the square models. To facilitate global re-scaling and then return the wire size and element spacing to their original values, I transferred the initial hex model to EZNEC. This program is also compatible with EZPlots, by AC6LA, from which I could make rapid graphs of gain, front-to-back ratio, feedpoint resistance and reactance, and SWR after making a frequency sweep across 2 meters at 0.25-MHz intervals. **Fig. 6** shows the final wire table, after all adjustments, for one of the test cases.

Note the X-coordinate in line 2. From zero to this point is the half-length of one side of the hexagon. Hence, 12 times the absolute value of the coordinate yields the circumference of the loop. It was thus possible to adjust the model for near resonance at 146 MHz and later return to find the resulting loop circumferences and what degree of adjustment was necessary relative to the square model. In all cases, element spacing and wire diameter are the same between models in a test sequence.



Wire	Seg.	X1	Y1	Z1	X2	Y2	Z2	Dia.	Conduct	Src/Ld
1	5	=H*A	0	=G*A	=G*A	0	=H*A	0.0625	Perfect	0/0
2	5	=G*A	0	=H*A	=-G*A	0	=H*A	0.0625	Perfect	0/0
3	5	=-G*A	0	=H*A	=-H*A	0	=G*A	0.0625	Perfect	0/0
4	5	=-H*A	0	=G*A	=-H*A	0	=-G*A	0.0625	Perfect	0/0
5	5	=-H*A	0	=-G*A	=-G*A	0	=-H*A	0.0625	Perfect	0/0
6	5	=-G*A	0	=-H*A	=G*A	0	=-H*A	0.0625	Perfect	0/0
7	5	=G*A	0	=-H*A	=H*A	0	=-G*A	0.0625	Perfect	0/0
8	5	=H*A	0	=-G*A	=H*A	0	=G*A	0.0625	Perfect	0/0
9	5	=H*B	=D	=G*B	=G*B	=D	=H*B	0.0625	Perfect	0/0
10	5	=G*B	=D	=H*B	=-G*B	=D	=H*B	0.0625	Perfect	0/0
11	5	=-G*B	=D	=H*B	=-H*B	=D	=G*B	0.0625	Perfect	0/0
12	5	=-H*B	=D	=G*B	=-H*B	=D	=-G*B	0.0625	Perfect	0/0
13	5	=-H*B	=D	=-G*B	=-G*B	=D	=-H*B	0.0625	Perfect	0/0
14	5	=-G*B	=D	=-H*B	=G*B	=D	=-H*B	0.0625	Perfect	1/0
15	5	=G*B	=D	=-H*B	=H*B	=D	=-G*B	0.0625	Perfect	0/0
16	5	=H*B	=D	=-G*B	=H*B	=D	=G*B	0.0625	Perfect	0/0
17	5	=H*C	=E	=G*C	=G*C	=E	=H*C	0.0625	Perfect	0/0
18	5	=G*C	=E	=H*C	=-G*C	=E	=H*C	0.0625	Perfect	0/0
19	5	=-G*C	=E	=H*C	=-H*C	=E	=G*C	0.0625	Perfect	0/0
20	5	=-H*C	=E	=G*C	=-H*C	=E	=-G*C	0.0625	Perfect	0/0
21	5	=-H*C	=E	=-G*C	=-G*C	=E	=-H*C	0.0625	Perfect	0/0
22	5	=-G*C	=E	=-H*C	=G*C	=E	=-H*C	0.0625	Perfect	0/0
23	5	=G*C	=E	=-H*C	=H*C	=E	=-G*C	0.0625	Perfect	0/0
24	5	=H*C	=E	=-G*C	=H*C	=E	=G*C	0.0625	Perfect	0/0

When we move to the octagon version of the 3-element quad, we need a different set-up for the 8 wires in each loop. **Fig. 7** shows the general set-up. You can correlate the variables A, B, and C to the focal line lengths calculated for the octagon to the values for the initially used loops with the same circumference in the square quad base-line model. Variables D and E designate the element spacing, counting from the reflector forward. Variables G and H simply list the sine and cosine of the relevant angles involved in calculating the corner coordinates. That angle was 30 degrees for the hexagon and is 22.5 degrees for the octagon. All octagon coordinates are functions of either the sine or the cosine of 22.5 degrees applied to the focal line length for each of the three loops. All of these variables for the octagon appear in a sample equations page in **Fig. 8**.



For ease of multiple-wire adjustments and some graphing via EZPlots, I once more transferred the model to EZNEC. The final values for one of the test models appear in **Fig. 9**. Note once more the X-coordinate in line 2 of the wire table. The absolute value of this coordinate is 1/2 the length of one side of the octagon. Hence, 16 times that value yields the circumference of the loop. As with the hexagon arrays, I brought each octagon array to the same near resonant condition as in the square array original, using the same element spacing and the same wire diameter. (Note that the screens used to illustrate the process do not necessarily represent the same test model.)

Wires

Wire Edit Other

Fig. 9

☐ Coord Entry Mode ☐ Preserve Connections

Final Values for a 3-Element Octagon Quad Beam

Wires

No.	End 1				End 2				Diameter	Segs
	X (in)	Y (in)	Z (in)	Conn	X (in)	Y (in)	Z (in)	Conn	(in)	
1	12.8886	0	5.33862	W8E2	5.33862	0	12.8886	W2E1	0.125	5
2	5.33862	0	12.8886	W1E2	-5.33862	0	12.8886	W3E1	0.125	5
3	-5.33862	0	12.8886	W2E2	-12.8886	0	5.33862	W4E1	0.125	5
4	-12.8886	0	5.33862	W3E2	-12.8886	0	-5.33862	W5E1	0.125	5
5	-12.8886	0	-5.33862	W4E2	-5.33862	0	-12.8886	W6E1	0.125	5
6	-5.33862	0	-12.8886	W5E2	5.33862	0	-12.8886	W7E1	0.125	5
7	5.33862	0	-12.8886	W6E2	12.8886	0	-5.33862	W8E1	0.125	5
8	12.8886	0	-5.33862	W7E2	12.8886	0	5.33862	W1E1	0.125	5
9	12.275	14.1928	5.08448	W16E2	5.08448	14.1928	12.275	W10E1	0.125	5
10	5.08448	14.1928	12.275	W9E2	-5.08448	14.1928	12.275	W11E1	0.125	5
11	-5.08448	14.1928	12.275	W10E2	-12.275	14.1928	5.08448	W12E1	0.125	5
12	-12.275	14.1928	5.08448	W11E2	-12.275	14.1928	-5.08448	W13E1	0.125	5
13	-12.275	14.1928	-5.08448	W12E2	-5.08448	14.1928	-12.275	W14E1	0.125	5
14	-5.08448	14.1928	-12.275	W13E2	5.08448	14.1928	-12.275	W15E1	0.125	5
15	5.08448	14.1928	-12.275	W14E2	12.275	14.1928	-5.08448	W16E1	0.125	5
16	12.275	14.1928	-5.08448	W15E2	12.275	14.1928	5.08448	W9E1	0.125	5
17	11.6787	32.2425	4.83748	W24E2	4.83748	32.2425	11.6787	W18E1	0.125	5
18	4.83748	32.2425	11.6787	W17E2	-4.83748	32.2425	11.6787	W19E1	0.125	5
19	-4.83748	32.2425	11.6787	W18E2	-11.6787	32.2425	4.83748	W20E1	0.125	5
20	-11.6787	32.2425	4.83748	W19E2	-11.6787	32.2425	-4.83748	W21E1	0.125	5
21	-11.6787	32.2425	-4.83748	W20E2	-4.83748	32.2425	-11.6787	W22E1	0.125	5
22	-4.83748	32.2425	-11.6787	W21E2	4.83748	32.2425	-11.6787	W23E1	0.125	5
23	4.83748	32.2425	-11.6787	W22E2	11.6787	32.2425	-4.83748	W24E1	0.125	5
24	11.6787	32.2425	-4.83748	W23E2	11.6787	32.2425	4.83748	W17E1	0.125	5
*										

A Sample Test Run

The 3-types of loops yield quads that look both similar and oddly different. **Fig. 10** provides to-scale outlines of each type of 3-element quad.

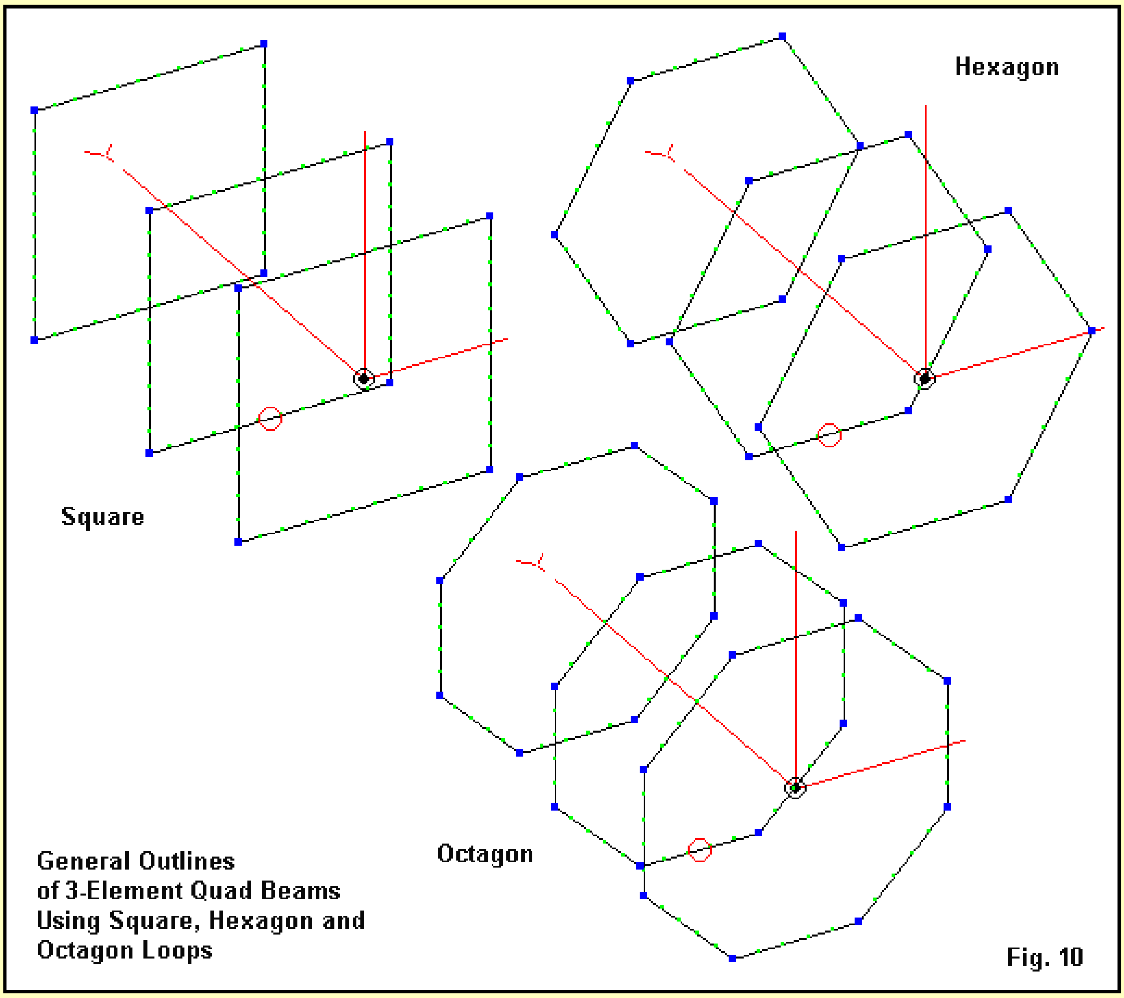


Fig. 10

Although the main elements of comparison among the square, hexagon, and octagon arrays are easily observable from data tables, it may be useful to do a bit of graphic analysis on at least one array as a guide to interpreting the tables. For this sample run, I shall use 0.125" diameter wire and look at the square and the octagon versions of the quad after optimizing the octagon version for resonance at 146 MHz.

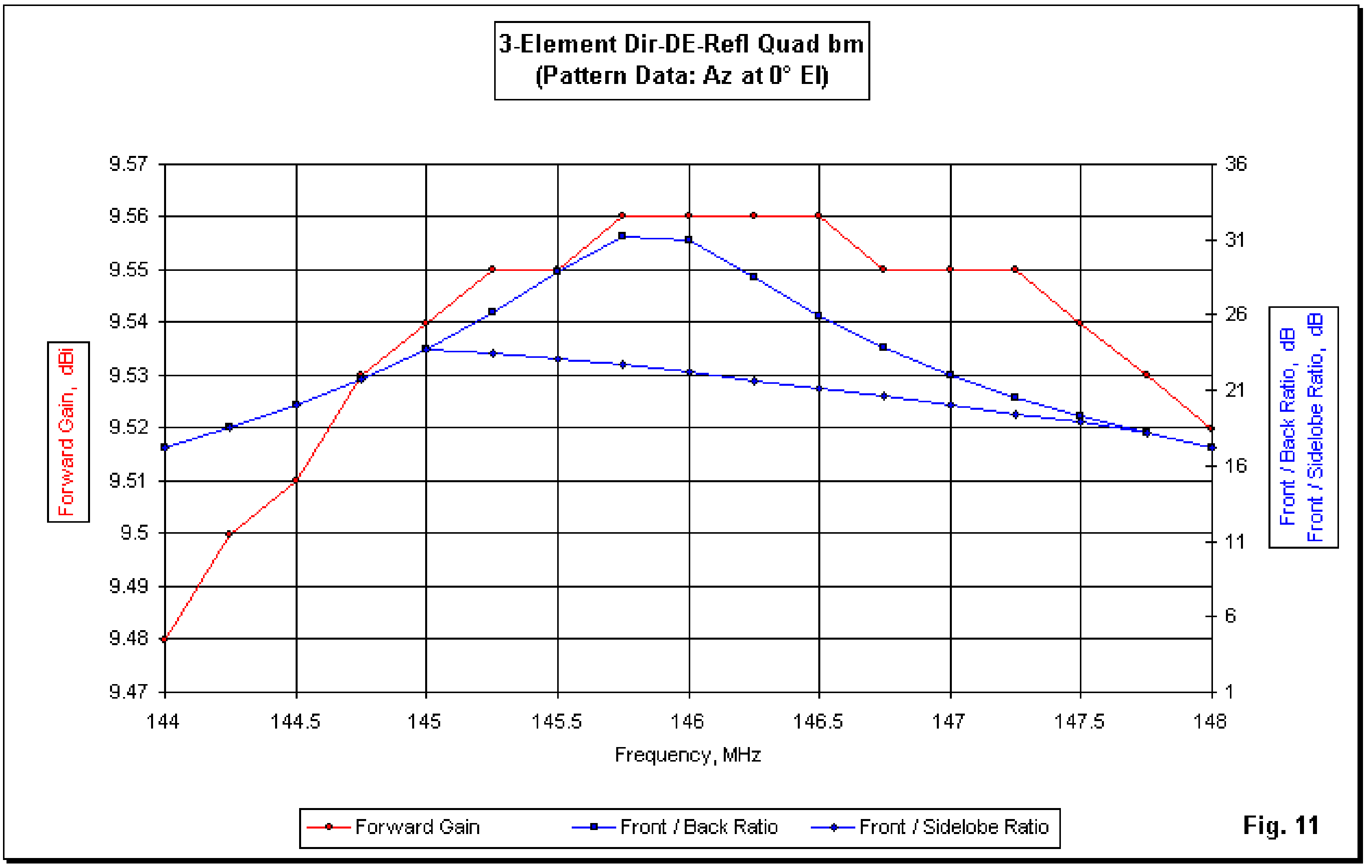


Fig. 11

Beginning with the square quad properties over the 2-meter band, we may look at the gain and front-to-back curves in **Fig. 11**. Note that the gain and front-to-back peaks are well centered in the passband. The front-to-sidelobe ratio is a measure of the worst-case front-to-back ratio for the array. It will always be a curve that either overlays the 180-degree front-to-back curve or departs from it in a shallower curve at somewhat lower values.

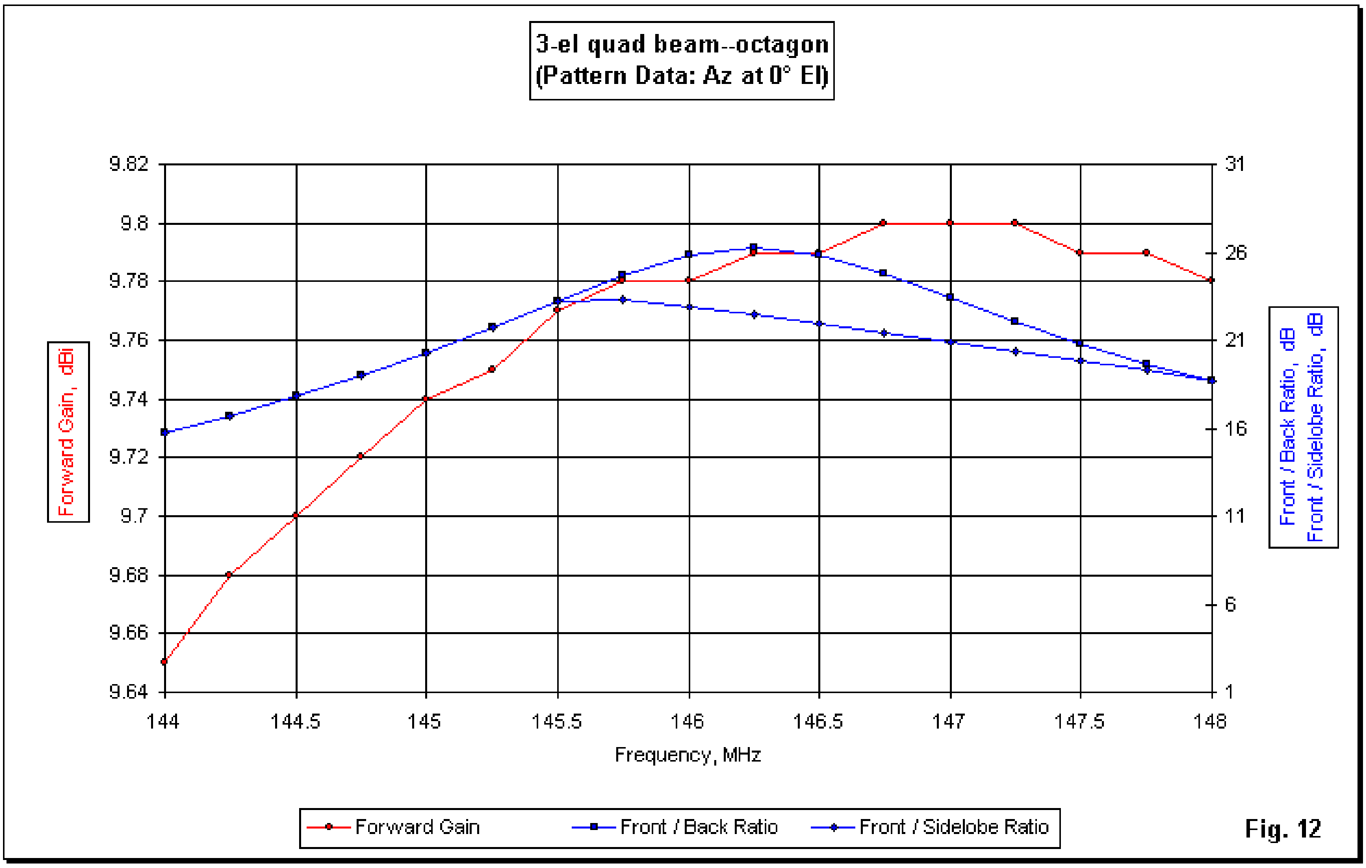


Fig. 12

Fig. 12 shows the comparable curve for the adjusted octagon array using the same wire size and element spacing. For the same resonant frequency, the octagon version shows a 180-degree front-to-back peak slightly above mid-band and a gain peak very much above mid-band. This pattern appears in all of the quasi-rounded versions of the arrays and indicates that changing shape has consequences for the performance curves.

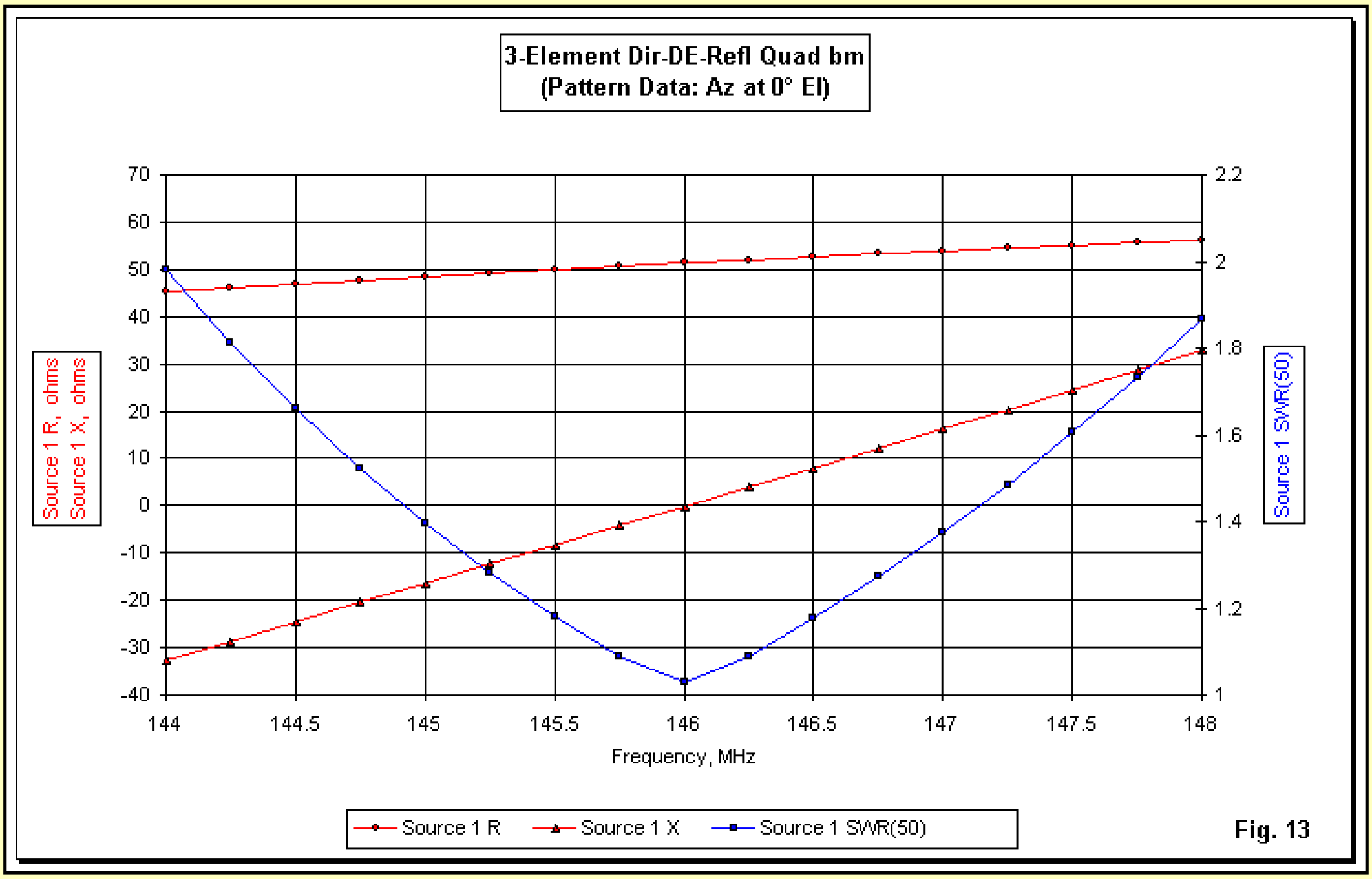


Fig. 13

Fig. 13 shows the original square quad with its near resonance at 146 MHz. Although we may trace with care the feedpoint resistance and reactance curves, we may use the band-edge 50-Ohm SWR as check points referenced to the right Y-axis scale. Now examine **Fig. 14** for the same points. This graph of feedpoint values is for the octagon version of the array. We note that the band-edge SWR values are higher than for the square quad, indicating that the quasi-rounded quads have slightly narrower passbands than the initial square version. As we examine the tabular results, we shall note that the narrowing passband applies to more categories of performance than just the 50-Ohm SWR.

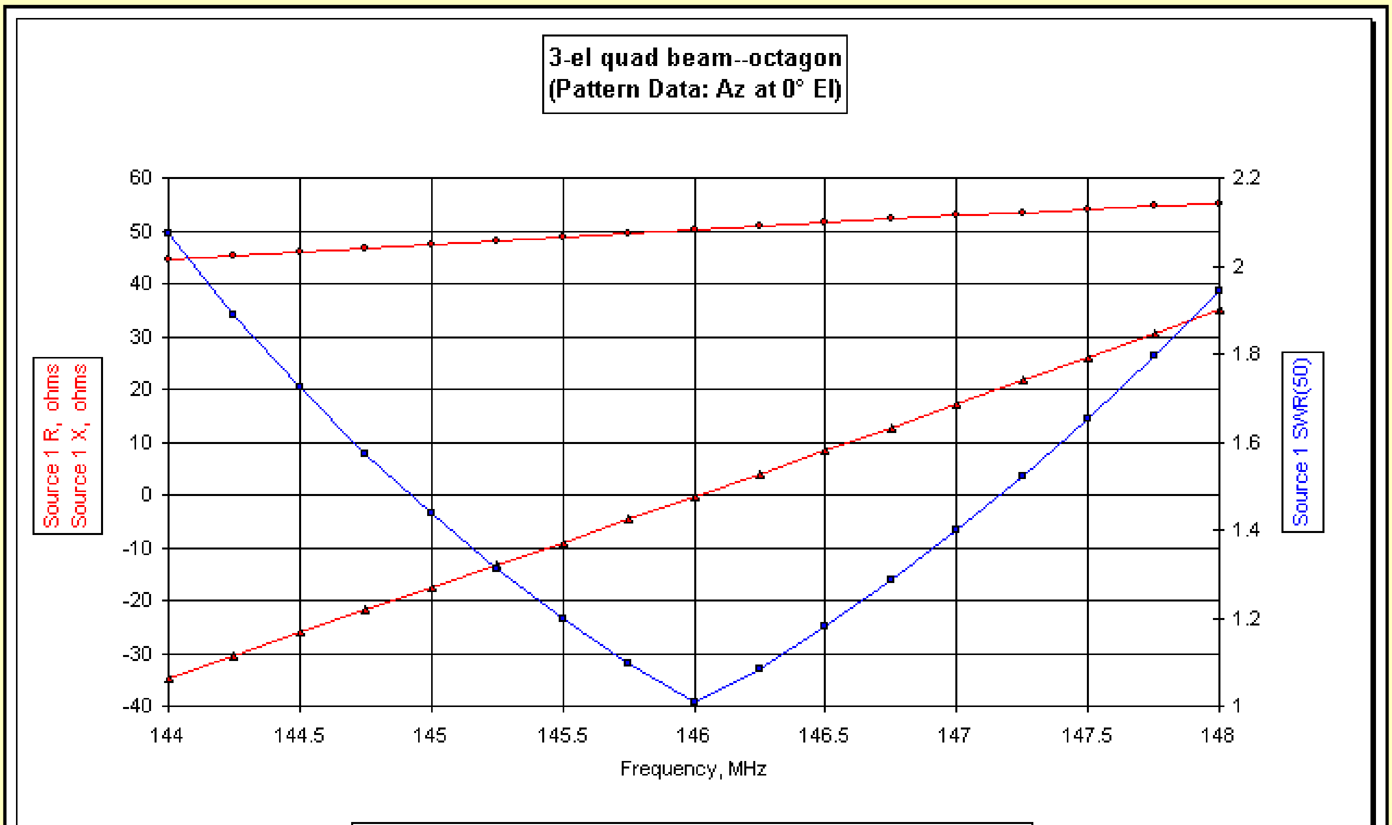


Fig. 14

The Tabulated Results

I created 2-meter 3-element quads with wire diameters of 0.25", 0.125" and 0.0625". The 2:1 wire size differentials provided a large wire-size range with a small number of samples. Each quad initially used a square version developed from the automated model program, with derived and adjusted hexagon and octagon versions. The models used perfect/lossless wire, although the differences between this selection and either copper or aluminum are not significant. Each derived model was adjusted until nearly resonant at 146 MHz, the near-resonant frequency of the original square model.

The following tables provide modeled free-space data at 144, 146, and 148 MHz for each of the models. Each model was subject to the Average Gain Test (AGT) to verify its adequacy. The worst value was 1.005, representing a maximum gain report error of 0.02 dB.

. . . .

0.25" Diameter Wire: 3-Element Quads

Square: AGT: 1.005 = 0.02 dB

Frequency	**144**	**146**	**148**
Gain dBi	**9.53**	**9.62**	**9.61**
180-deg F-B dB	**18.27**	**30.10**	**18.68**
Feed Z (R+/-jX Ohms)	**44.5 - j29.0**	**49.7 - j 0.7**	**54.1 + j28.6**
50-Ohm SWR	**1.851**	**1.015**	**1.732**

Hexagon: AGT: 1.005 = 0.02 dB

Gain dBi	**9.66**	**9.78**	**9.79**
180-deg F-B dB	**17.14**	**26.50**	**19.57**
Feed Z (R+/-jX Ohms)	**44.2 - j29.7**	**49.2 - j 0.3**	**53.7 + j30.5**
50-Ohm SWR	**1.884**	**1.017**	**1.794**

Octagon: AGT: 1.004 = 0.02 dB

Gain dBi	**9.69**	**9.82**	**9.85**
180-deg F-B dB	**16.64**	**25.13**	**19.88**
Feed Z (R+/-jX Ohms)	**44.0 - j30.6**	**48.9 - j 0.7**	**53.3 + j30.5**
50-Ohm SWR	**1.919**	**1.027**	**1.796**

0.125" Diameter Wire: 3-Element Quads

Square: AGT: 1.004 = 0.02 dB

Frequency	144	146	148
Gain dBi	9.48	9.56	9.52
180-deg F-B dB	17.26	30.91	17.24
Feed Z (R+/-jX Ohms)	45.2 - j32.8	51.4 - j 0.2	56.2 + j33.1
50-Ohm SWR	1.980	1.028	1.868

Hexagon: AGT: 1.003 = 0.01 dB

Gain dBi	9.61	9.73	9.72
180-deg F-B dB	15.95	27.10	18.52
Feed Z (R+/-jX Ohms)	44.7 - j34.9	50.6 - j 0.9	55.5 + j34.0
50-Ohm SWR	2.074	1.022	1.901

Octagon: AGT: 1.002 = 0.01 dB

Gain dBi	9.65	9.78	9.78
180-deg F-B dB	15.70	25.79	18.70
Feed Z (R+/-jX Ohms)	44.7 - j34.9	50.4 - j 0.6	55.3 + j35.0
50-Ohm SWR	2.078	1.013	1.935

0.0625" Diameter Wire: 3-Element Quads

Square: AGT: 1.003 = 0.01 dB

Frequency	144	146	148
Gain dBi	9.42	9.50	9.43
180-deg F-B dB	16.31	31.58	16.07
Feed Z (R+/-jX Ohms)	45.8 - j37.0	53.1 - j 0.4	58.3 + j36.7
50-Ohm SWR	2.138	1.062	1.981

Hexagon: AGT: 1.002 = 0.01 dB

Gain dBi	9.56	9.69	9.65
180-deg F-B dB	15.36	28.49	17.14
Feed Z (R+/-jX Ohms)	45.5 - j38.3	52.3 - j 0.1	57.8 + j38.9
50-Ohm SWR	2.199	1.046	2.060

Octagon: AGT: 1.002 = 0.01 dB

Gain dBi	9.60	9.74	9.71
180-deg F-B dB	14.85	26.70	17.63
Feed Z (R+/-jX Ohms)	45.3 - j39.7	51.9 - j 0.9	57.3 + j38.7
50-Ohm SWR	2.264	1.043	2.053

. . . .

1. Changes in Performance: For each wire size, we can observe 2 phenomena. First, the operating passband in each category of performance narrows a small amount as we move from the square to the hexagon to the octagon. Second, the peak performance shifts upward in frequency relative to the resonant frequency of the array.

The most likely reason for these changes in performance as we gradually round the shape of the quad loops is that the mutual coupling between elements also changes. Remember that we scaled the loops but retained the spacing of the initial square-quad version.

Therefore, although a quad adapted from a square to an octagon (and, by extension, a circle) may provide satisfactory performance for a given design passband, further optimizing to center the performance peaks at the design frequency is both possible and recommended.

2. Changes in Peak Gain: As we gradually round the corners of a quad, we see an increase in peak gain. From a square to an octagon, the rise is about 0.24 dB for each of the wire sizes. about two-thirds of the rise occurs in the shift from a square to a hexagon. Hence, the further rise from an octagon to a circle is only likely to create a total gain increase of 0.3 dB over a square quad.

Whether the gain increase--assuming that one also re-optimizes the array to place performance peaks at the design frequency--is justified is a builder/user decision, most likely based upon the degree of difficulty in the re-design and the construction of the circular elements. A 0.3-dB change of forward gain would not normally be detectable in array operation, since it is just about 5% of 1 S-unit.

The following table provides element circumferences and element spacing for the arrays whose performance data we have just observed. All dimensions are in inches. Element spacing is listed only once for the square loop, since it is the same for all 3 arrays in each set.

. . . .

0.25" Diameter Wire: 3-Element Quads

Element or Spacing	Square	Hexagon	Octagon
Reflector	88.552	86.793	86.070
Driver	83.749	82.085	81.401
Director	79.243	77.669	77.022
Reflector-Driver Spacing	14.150		
Reflector-Director Spacing	32.174		
Circumference Ratio Relative to Square Loops		0.9801	0.9720

0.125" Diameter Wire: 3-Element Quads

Element or Spacing	Square	Hexagon	Octagon
Reflector	87.515	85.957	85.359
Driver	83.349	81.865	81.296
Director	79.300	77.888	77.347
Reflector-Driver Spacing	14.193		
Reflector-Director Spacing	32.243		
Circumference Ratio Relative to Square Loops		0.9822	0.9754

0.0625" Diameter Wire: 3-Element Quads

Element or Spacing	Square	Hexagon	Octagon
Reflector	86.739	85.402	84.840
Driver	83.042	81.761	81.222
Director	79.337	78.114	77.599
Reflector-Driver Spacing	14.244		
Reflector-Director Spacing	32.277		
Circumference Ratio Relative to Square Loops		0.9846	0.9781

. . . .

As we increase the wire diameter, the required amount of circumference shortening increases. To determine if the increase was predominantly a function of wire size or whether array design played a significant role on the adjustment factor, I turned to a second 3-element array for which there is an automated design model and program. The wide-band design calls for dimensions significantly different from those of the high gain design so that a variation ought to appear in the correction factor, at least for the octagon version. Therefore, I created a wide-band model using 0.0625" diameter lossless wire for a square-loop array centered at 146 MHz. I then re-calculated the array for octagon loops and adjusted it for near-resonance at 146 MHz, using the same procedure as on the previous arrays.

The following tables summarize the performance data and the dimensions of square and octagon versions of the wide-band quad beam. Since the feedpoint impedance is close to 70 Ohms, both 50-Ohm and 75-Ohm SWR values are listed.

. . . .

0.0625" Diameter Wire: 3-Element Wide-Band Quads

Square: AGT: 1.003 = 0.01 dB

Frequency	144	146	148
Gain dBi	9.11	8.99	8.78

180-deg F-B dB	**15.78**	**34.02**	**17.76**
Feed Z (R+/-jX Ohms)	**55.7 - j37.2**	**72.6 - j 0.6**	**88.6 + j31.6**
50-Ohm SWR	**2.012**	**1.452**	**2.081**
75-Ohm SWR	**1.891**	**1.034**	**1.520**

Octagon: AGT: 1.002 = 0.01 dB

Gain dBi	**9.33**	**9.29**	**9.12**
180-deg F-B dB	**14.07**	**26.01**	**20.06**
Feed Z (R+/-jX Ohms)	**54.8 - j40.3**	**70.8 - j 1.0**	**87.3 + j34.6**
50-Ohm SWR	**2.132**	**1.416**	**2.121**
75-Ohm SWR	**1.992**	**1.061**	**1.568**

. . . .

0.0625" Diameter Wire: 3-Element Wide-Band Quads

Element or Spacing	**Square**	**Octagon**
Reflector	**87.086**	**85.179**
Driver	**82.320**	**80.517**
Director	**75.922**	**74.260**
Reflector-Driver Spacing	**13.155**	
Reflector-Director Spacing	**35.914**	
Circumference Ratio Relative to Square Loops		**0.9781**

. . . .

The octagon version of the wide-band array shows a variable increase of gain over the square version, because the true gain peak of the square version lies just below the lower limit of the operating passband. However, a peak increase of about 0.3 dB appears possible. As with the high gain design, we note a slight narrowing of the operating passband as we round the square into an octagon.

Since the octagon circumference ratio relative to the square version is identical for both the high gain and the wide-band designs to 4 decimal places, it is likely that we may ascribe the changes in the ratio values to the wire size changes alone.

From Octagon to Circle

The circumference ratios derived from the exercise are listed to 4 decimal places as a simple function of the calculations involved. It is unlikely that any backyard builder would be able to use values of more than 2 significant figures. In general, then, if the wire size is greater than about 0.1", a correction factor of 0.97 is adequate and for wires 0.1`" and less, 0.98 is adequate.

However, these values apply only so far as we round a square to an octagon. If we return to **Fig. 2**, we shall find that an octagon has a ratio of focal line to circumference that is only about 3/4 of the way from the corresponding ratios for a square and for a circle. Further extrapolation may be necessary to arrive at factors applicable to a circle, if we use the square loop as a starting point.

It is therefore likely that for wire sizes greater than about 0.1" diameter, a final correction factor of 0.96 would apply, and for wires sizes less than 0.1" diameter, a correction factor of 0.97 would be closer to the mark--when moving all the way from a square quad to its circular counterpart. (See the **Appendix** for a further refinement of these numbers based upon a 16-sided loop model.)

The design considerations that we have observed from the compilation of performance data place a further caution on the application of any such correction factors. The movement of the peaks in the performance curves as we made the transition from a square through a hexagon to an octagon strongly suggest that a circular design requires careful optimization to return those peaks to the vicinity of the design frequency. This process may result in changes to the element spacing and the loop circumferences. Hence, the correction factor may turn into different numbers for each of the loops in the array, plus a pair for the element spacing.

As a simple example of further optimization, we can restore the position of the front-to-back ratio peak by scaling the element spacing as well as the loop sizes once we discover where the initial octagon's resonant frequency is. However, as shown in **Fig. 15**, the gain peak remains high in the passband (even though the total gain change across the band is only 0.13 dB). This model is a further modification of the high gain quad with 0.25" diameter elements. On balance, relative to our earlier octagon model, we give up a tiny bit of gain but acquire a better bandwidth for higher front-to-back ratios.

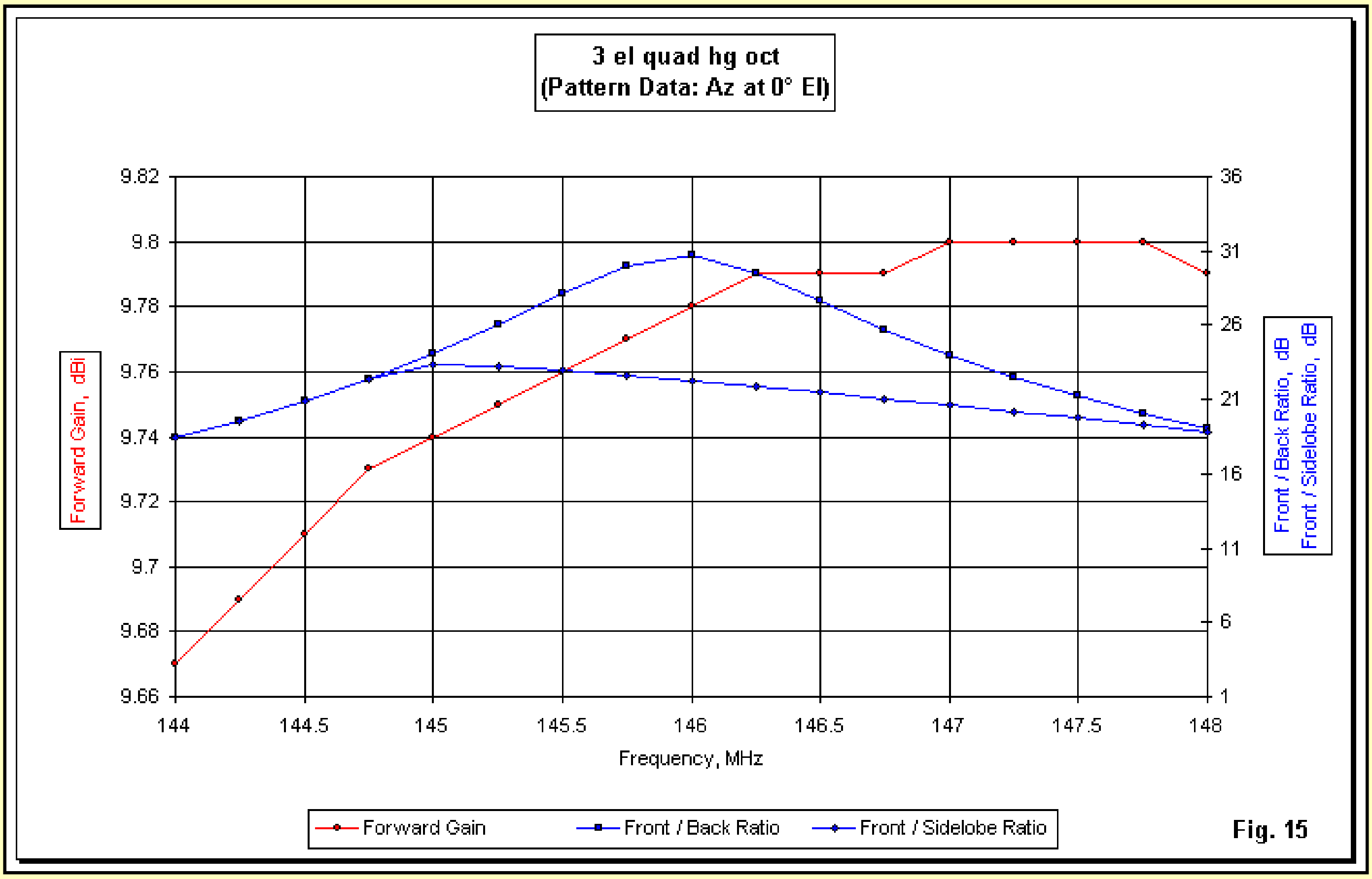


Fig. 15

As well, the resistance, shown in the feedpoint graph in **Fig. 16**, reaches 50 Ohms well above the mid-band point, suggesting slight changes in the size or spacing of the reflector. However, these changes will create changes in the other curves that we are trying to balance in a perfectly optimized quad array.

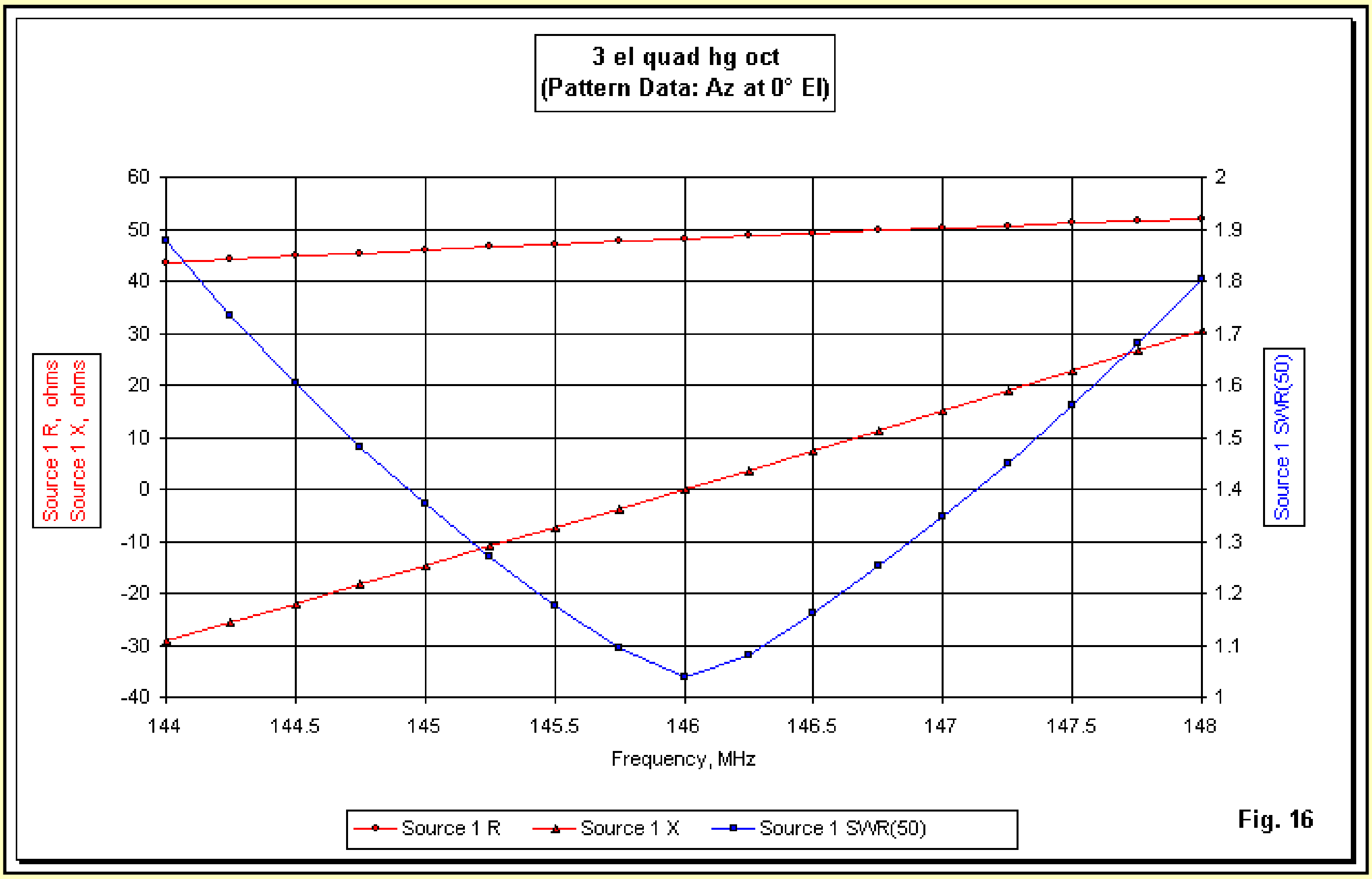


Fig. 16

For reference, the following tables present a comparison of the performance and the dimensions of the 0.25" element octagons under our initial assumptions and with the further refinements of element spacing.

. . . .

3-Element High Gain Octagon Quad Performance: 0.25" Diameter

With Square Version Element Spacing

Frequency MHz	**144**	**146**	**148**
Gain dBi	**9.69**	**9.82**	**9.85**
180-deg F-B dB	**16.64**	**25.13**	**19.88**
Feed Z (R+/-jX Ohms)	**44.0 - j30.6**	**48.9 - j 0.7**	**53.3 + j30.5**
50-Ohm SWR	**1.919**	**1.027**	**1.796**

With Scaled Element Spacing

Frequency MHz	**144**	**146**	**148**
Gain dBi	**9.67**	**9.78**	**9.79**
180-deg F-B dB	**18.39**	**30.67**	**19.08**
Feed Z (R+/-jX Ohms)	**43.6 - j29.2**	**48.2 - j 0.0**	**52.0 + j30.5**
50-Ohm SWR	**1.876**	**1.038**	**1.805**

. . . .

3-Element High Gain Octagon Dimensions: 0.25" Diameter

Element or Spacing	**Square Spacing**	**Scaled Spacing**
Reflector	**86.070**	**86.185**
Driver	**81.401**	**81.510**
Director	**77.022**	**77.125**
Reflector-Driver Spacing	**14.150**	**13.772**
Reflector-Director Spacing	**32.174**	**31.314**

. . . .

These maneuvers, of course, apply to the octagon. Given the changes in the curves as we moved from the square to the octagon, we can expect further movement in the same directions as we approach a true circle for the quad loops. Hence, optimization of the octagon is an indicator of, but not a guarantee of, the performance of a quad with perfectly circular loops.

Anyone who wishes to design a true circular element quad with NEC or MININEC software should therefore begin with a polygon of at least 12 to 16 sides. Although the dimension shown in this exercise form a starting point for the design, they will require careful re-sizing to arrive at a truly satisfactory design that assures that we obtain the full 0.3 dB gain over a square that circular loops offer at the frequency on which we wish to have that gain.

Appendix: Going to 16-Sided Loops

After adjusting the element spacing for the 8-sided loop to restore some of the performance curves as we round the square loop quad beam, I decided to create a 16-sided loop version of the array. The techniques for generating a 16-sided loop are an extension of those used to create the octagon, so we may by-pass them and go straight to the results. These results will be based on adjusting or re-scaling the element spacing along with the loop circumference as we bring the array to near resonance at 146 MHz.

As suspected, the 16-sided version of the array required further adjustment beyond the 8-sided loop version. The following table provides the dimensions in parallel columns for the initial square version, the octagon, and the added 16-sided version, all using lossless 0.25"-diameter wire.

. . . .

3-Element High Gain Octagon Dimensions: 0.25" Diameter

Element or Spacing	4 sides	8 sides	16 sides
Reflector	88.552	86.185	85.418
Driver	83.749	81.510	80.784
Director	79.243	77.125	76.438
Reflector-Driver Spacing	14.150	13.772	13.649
Reflector-Director Spacing	32.174	31.314	31.035
Adjustment Factor Relative to Square Loops		0.9733	0.9646

. . . .

A 16-sided loop has a focal-line-to-circumference ratio of 0.1602 compared to a circle's 0.1592. The inverse values are 6.2832 for the circle and 6.2429 for the 16-sided loop. The 16-sided loop is close enough to a circle that we may reaffirm the recommended correction factors developed only by going to an 8-sided loop model. ***When moving from a square to a circle, for wires fatter than about 0.1" diameter, multiply all dimensions (that is, circumferences and element spacing) by about 0.96. For wires less than about 0.1", use a multiplier of 0.97.*** These values tend to confirm the extrapolation performed on the basis of the octagon version of the quad. However, we now strongly suggest that the builder apply the adjustments to the element spacing as well as the element circumference.

The modeled free-space performance of the 16-sided model appears in the following table.

. . . .

3-Element High Gain 16-Sided Quad Performance: 0.25" Diameter

Frequency MHz	144	146	148
Gain dBi	9.67	9.78	9.80
180-deg F-B dB	18.39	31.26	19.19
Feed Z (R+/-jX Ohms)	44.0 - j30.1	48.1 - j 0.4	51.4 + j30.5
50-Ohm SWR	1.901	1.040	1.809

. . . .

As the table suggests, the gain curve continues to move higher in the passband as we further round the corners of the loops. However, it is possible to use the adjustment factor to center the front-to-back curves, as shown in **Fig. 17**.

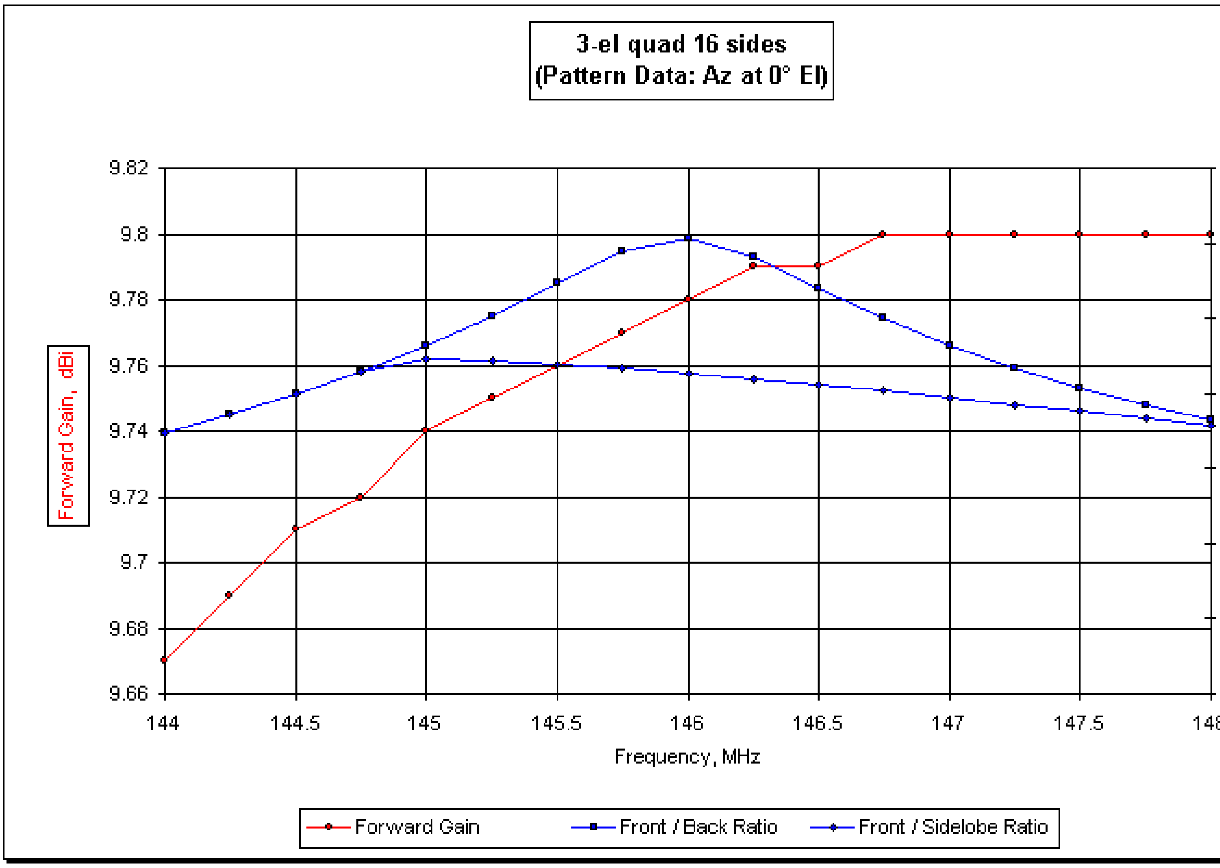


Fig. 17

The form of adjustment developed here also allows the SWR curve to remain well centered in the passband, as revealed in **Fig. 18**. However, as we saw in the move through the hexagon and the octagon, the operating passband continues to narrow. As well, the resistive component of the feedpoint impedance continues to slowly decrease. However, nothing in the progression of charts suggests that a true circular loop version of the array would not--using 0.25" diameter elements--meet a 2:1 50-Ohm SWR limit at the band edges.

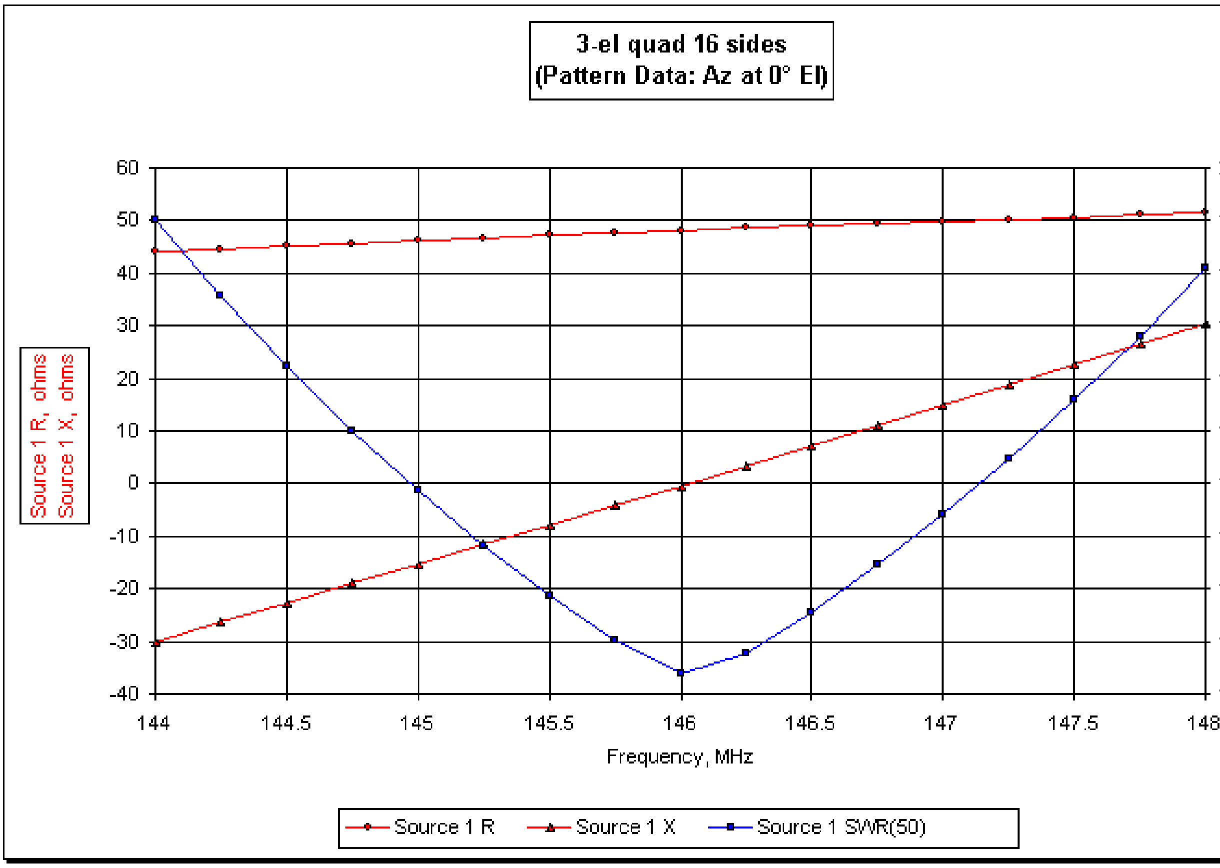


Fig. 18

The bottom line, then, is that we may adjust a square quad to a circular loop quad by adjusting both the loop circumferences and element spacing. However, we should expect a slight reduction in the operating bandwidth. As well, the design might well use further optimizing to return the gain peak to the design frequency.

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